Teaching ideas for Option E, *Astrophysics*

Questions

A number of worksheets are provided for this Option:

- support questions examine the very basic concepts of the syllabus
- extended questions delve deeper and are equivalent to exam level questions.

Teaching ideas

- The magnitude scale is confusing to students when they first meet it, so it is important to give many examples to make sure students understand that the smaller the magnitude the brighter the star appears to be.
- Students will always ask where the letters OBAFGKM for spectral classes come from. There is no interesting story behind this though; they started as A, B, etc. but were later reordered.
- Students find the story of the Eddington/Chandrasekhar controversy about collapsed stars interesting and it is worth spending some time on it. Freeman Dyson's obituary of Chandrasekhar, in *Nature*, **438**, 1086 (22 December 2005), *The death of a star*, is very good.

Case study: An elementary derivation of the mass–luminosity relation

The mass–luminosity relation is an approximate relation that links the luminosity of a star and its mass. It was first derived by A. Eddington in the early part of the 20th century. It only applies to stars on the main sequence. It is amazing that very simple physics can lead to this relation without detailed knowledge of the nuclear fusion reactions taking place inside the star or any other details about the structure of the star.

All we have to assume is that the star is in equilibrium under the action of the force of gravitation that tends to collapse the star and the outward force due to the pressure inside the star. The big assumption is to say that the material of the star obeys the ideal gas law. This of course is not a very good assumption in general, but main sequence stars that are not too massive or too light obey this law approximately. The gas law states that $PV = nRT$. The number of moles is equal to $n = \frac{N}{N_A}$, where N_A is Avogadro's number. The Boltzmann constant k is defined by $k = \frac{R}{N_A}$ and so the gas law may be written as $PV = NkT$. Since $V = \frac{4\pi R^3}{3} \propto R^3$ and $N \propto M$, we have that $PR^3 \propto MT$. The force on a patch on the surface of the star of area A is dP A where dP is the difference in pressure at the faces of the patch. The gravitational force is $G\frac{m(r)\mathrm{d}m}{r^2}$ where dm is the infinitesimally small mass in the patch. This mass is given by dm = volume × density =ρA dr and so the gravitational force on the patch is $G\frac{\rho m(r)A\mathrm{d}r}{r^2}$. To have equilibrium, the gravitational force on the shell toward the centre must be balanced by the outward force due to the pressure in the star's interior. This means that $\frac{G\rho m(r)A\mathrm{d}r}{r^2} = A\mathrm{d}P$ or $\frac{\mathrm{d}P}{\mathrm{d}r} = \frac{G\rho m(r)}{r^2}$. Assuming a constant pressure inside the star and

$\frac{P}{R} = \frac{G\rho M}{R^2} = \frac{G\frac{M}{\frac{4}{3}\pi R^3}M}{R^2} = \frac{3GM^2}{4\pi R^5}$, i.e. $P = \frac{3GM^2}{4\pi R^4}$, i.e. $P \propto \frac{M^2}{R^4}$. Combining $PR^3 \propto MT$ with the equation just derived gives $T \propto \frac{M}{R}$. This shows that as the star shrinks (i.e. the radius gets smaller) the temperature goes up. (Since the temperature goes up, the internal energy of the star increases. This energy comes from the gravitational potential energy of the star that is released as the star shrinks.)

Now, the luminosity is given by $L = \sigma A T^4 \propto R^2 \left(\frac{M}{R}\right)^4 = \frac{M^4}{R^2}$. And since $\rho \propto \frac{M}{R^3}$ i.e. $R \propto \left(\frac{M}{\rho}\right)^{1/3}$ it follows that $L \propto \frac{M^4}{M^{2/3}} = M^{3.3}$, the mass–luminosity relation!

Applications of the mass–luminosity relation

- The mass–luminosity relation has many applications. The first is to determine whether a given star whose luminosity is given in terms of the Sun's luminosity is in fact a main sequence star.
- The second application is to determine the time a star will spend on the main sequence. In rough terms the energy available for fusion is the entire mass *M* of the star. The energy that theoretically can be released is then $E = Mc^2$. Since $L = \frac{E}{T}$, it follows that $kM^{3.3} = \frac{Mc^2}{T}$, i.e. $T \propto \frac{1}{M^{2.3}}$. So for a star of mass double that of the Sun the lifetime *T* is *less* than that of the sun by a factor of $2^{2.3} \approx 4.9$.

Practical activities/ICT

- The site http://outreach.atnf.csiro.au/education/senior/astrophysics/ is a wonderful site with all kinds of activities.
- For many useful activities related to the H-R diagram see http://www.astro.washington.edu/courses/labs/clearinghouse/homeworks/hrdiagram.html
- For activities related to stellar properties and stellar evolution see http://www.unm.edu/%7Eastro1/101lab/lab8/lab8.html
- For good quality explanations and materials, including practical activities, see Cornell University's site for Astronomy: http://www.astro.cornell.edu/app/webroot/academics/courses/astro201/topics.html
- http://www.astronomy.ohio-state.edu/~pogge/Ast162/#lectures
- For an incredible series of materials and practical exercises, see the European Space Agency's website at http://www.astroex.org/
- Very many interesting activities for every part of the course can be found here: http://astro.unl.edu/naap/
- To look at eclipsing binaries, where you can change temperature and star radii, try: http://www.compadre.org/osp/search/search.cfm?gs=222&b=1&qc=Compiled%20Simulation

Common problems

- As mentioned above, students often have problems realising that the dimmer the star appears the greater the apparent magnitude. It may be worth mentioning that $m = A - \frac{5}{2}\log b$, where *A* is a constant. This then explains that $m_1 - m_2 = A - \frac{5}{2}B\log b_1 - (A - \frac{5}{2}\log b_2) = -\frac{5}{2}\log\frac{b_1}{b_2}$ so that $\frac{b_1}{b_2} = 2.51^{m_2 - m_1}$ a result that all students must be able to know and use (notice that $2.51 \approx \sqrt[5]{100}$).

Theory of knowledge (TOK)

- The study of astrophysics makes it clear that physics has managed to ask fundamental questions about the Universe and its future evolution and eventual fate, questions that have been on man's mind since ancient times. Unlike the ancients though, modern astrophysics has been able to provide precise methods that are helping answer these questions. Much of the success of modern astrophysics comes from a combination of theoretical/mathematical advances and models along with very substantial technological advances that have made possible measurements and observations that only a few years ago seemed impossible.
- Note that the scientific community takes the concept of dark energy very seriously even though no one has any clear idea of what that energy might be.